Static Pressure-Velocity Field in a Turbulent Jet

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Introduction

Velocity fluctuations have attracted the main attention when the kinetic mechanism of turbulent flow is discussed, although static pressure fluctuations are another important turbulence quantity. Both kinds of fluctuations closely interact with each other in turbulent flow, and the static pressure-velocity correlation is an indispensable factor in clarifying the turbulent transport mechanism of mechanical energy. The static pressure fluctuation is an important factor for the turbulent breaking-up process in the mixing of nonmiscible fluids, movement of small particles in turbulent flow, production of noise, and so forth. The quantitative relation between static pressure fluctuations and velocity fluctuations remains uncertain from an experimental viewpoint, although there have been a few papers concerning measurements of static pressure fluctuations inside flowing fluids (Elliott, 1972; Hammersley and Jones, 1975; Jones et al., 1979; Komerath et al., 1983; Nithianandan et al., 1979).

The purpose of this study is to clarify experimentally the local interaction between static pressure fluctuations and velocity fluctuations in a turbulent jet of an incompressible fluid. This is done using a new method (Kuroda et al., 1981) of simultaneous measurements of both fluctuations in turbulent liquid flow, which was proposed by applying the electrochemical technique (Ito et al., 1974) for measuring three-dimensional velocity fluctuations.

Experimental Method

A probe for measuring static pressure fluctuations and three-dimensional velocity fluctuations of liquid is shown in Figure 1. Four point-electrodes and a small pressure hole are arranged on an acrylic resin sphere, and the instantaneous velocity U and the instantaneous flow angle θ between the velocity vector and the OM axis of the probe can be measured by using an electrochemical method (Ito et al., 1974). The instantaneous pressure P at the pressure hole is measured by a pressure transducer set just under the hole, with as short a tube for pressure transmission as possible. When the potential flow is assumed around the sphere, the relation between P and the static pressure P_s of the surrounding flow is expressed by the following equation in the range of about $\theta < \pi/6$ rad:

$$P_s = P - (1/2)\,\rho U^2\,[1 - (9/4)\sin^2\theta] \tag{1}$$

The value of P_s can be calculated instantaneously by using Eq. 1.

The test fluid used was an aqueous solution containing two electrolytes, $K_3Fe(CN)_6$ and $K_4Fe(CN)_6$, each 3.0×10^{-3} mol/L, and with KCl (1.0×10^{-1} mol/L) to act as the supporting electrolyte. The test fluid was injected into a 312 mm ID cylindrical tank through a 10 mm ID round jet nozzle that was set at the center of the bottom. Experiments were made in the range of $5{,}000 \leqq Re(=U_o d/\nu) \leqq 11{,}000$, where U_o is the velocity of the primary stream at the jet nozzle and d is the inner diameter of the jet nozzle.

Results and Discussion

Turbulent Euler number

The relationship between static pressure fluctuations p_s and velocity fluctuations u_i in incompressible Newtonian fluids is expressed by the following equation (Rotta, 1972):

$$\frac{\partial^2 p_s}{\partial x_i \partial x_i} = -2\rho \frac{\partial \overline{U}_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} - \rho \frac{\partial^2 (u_i u_j)}{\partial x_i \partial x_j} + \rho \frac{\partial^2 \overline{u_i u_j}}{\partial x_i \partial x_j} \tag{2}$$

The following characteristic parameters are introduced with reference to the idea of general mixing-length theory (Goldstein, 1965):

- Intensity of static pressure fluctuation p_s: $p'_s = (\overline{p_s^2})^{1/2}$
- Intensity of velocity fluctuation u_i: $u' = [(\overline{u_i u_i})/3]^{1/2}$
- Turbulent characteristic length (mixing length): ℓ
- Intensity of mean velocity gradient: $S = (e_{ij} e_{ij})^{1/2}$ (e_{ij} is a mean strain rate tensor).

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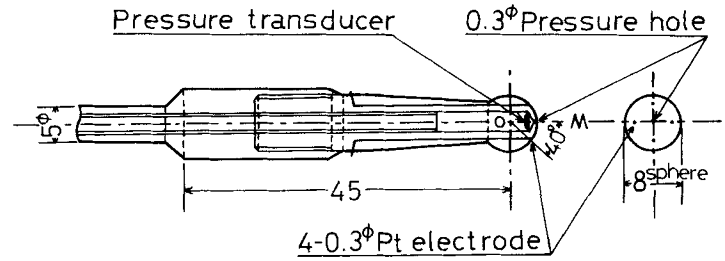

Figure 1. Diagram of probe.

By using these characteristic parameters, Eq. 2 is rewritten in a dimensionless form as follows,

$$\frac{p_s'}{\rho u'^2}\frac{\partial^2 p_s^*}{\partial x_i^* \partial x_i^*} = -2\frac{Sl}{u'}\left(\frac{\partial \overline{U}_i}{\partial x_j}\right)^* \frac{\partial u_j^*}{\partial x_i^*} - \frac{\partial^2 (u_i^* u_j^*)}{\partial x_i^* \partial x_j^*} + \frac{\partial^2 \overline{u_i^* u_j^*}}{\partial x_i^* \partial x_j^*} \tag{3}$$

where * denotes the dimensionless variable. This equation involves two dimensionless quantities, $p_s'/(\rho u'^2)$ and $S\ell/u'$. The former is regarded as a kind of Euler number based on turbulent fluctuations and is symbolized as Eu_t:

$$Eu_t = p_s'/(\rho u'^2) \tag{4}$$

When an idea of the mixing length theory for one-dimensional turbulent diffusion—that is, the proportional relationship between the velocity fluctuation and the mean velocity gradient multiplied by the mixing length—is assumed to be applicable also in this three-dimensional study, the following relation is expected.

$$u' \propto \ell \cdot S \tag{5}$$

and therefore the dimensionless quantity $S\ell/u'$ is considered to become constant. Here, if the similarity of turbulence structure appears in the jet mixing region by using the above-mentioned characteristic parameters, Eu_t is also expected to be constant at any position in the jet mixing region. In this study, Eu_t is investigated experimentally.

Figure 2 shows distributions of p_s' at $Re = 11{,}000$ and the similarity of distributions can be seen, where $\overline{U}_{zm}$ is the maximum value of $\overline{U}_z$ at the center axis. The distance of the virtual source of the jet from the nozzle exit is $a(= -3.0 \times 10^{-2}$ m). Similar distributions hold also for the axial velocity $\overline{U}_z$ and the turbulent intensities $\overline{u_i^2}$, and the flow condition in the measured region is considered to be fully turbulent. In such conditions, Eu_t takes a constant value of 4.3 in all the measured positions, as shown in Figure 3. On the other hand, Eu_t changes for the smaller value of Re or the larger value of z. This tendency may be natural when the turbulent condition in the measured region changes from the fully turbulent condition to the transitional condition. Figure 4 shows a relationship between $Re \cdot d/(z + a)$ and Eu_t at the center axis, and it indicates that the fully turbulent condition is found in the range of $Re \cdot d/(z + a) > 250$.

Static pressure-velocity correlation

The turbulent static pressure-velocity correlation $\overline{p_s u_i}$ has an important function in the transport equation of turbulence energy and is considered to express the turbulent transport flux of static pressure enrgy. If the turbulent field is assumed to spread infinitely, the turbulent static pressure-velocity correla-

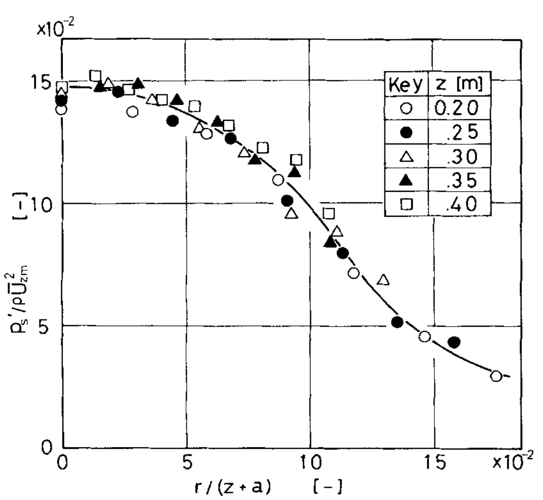

Figure 2. Similarity of p_s' distribution at Re = 11,000.

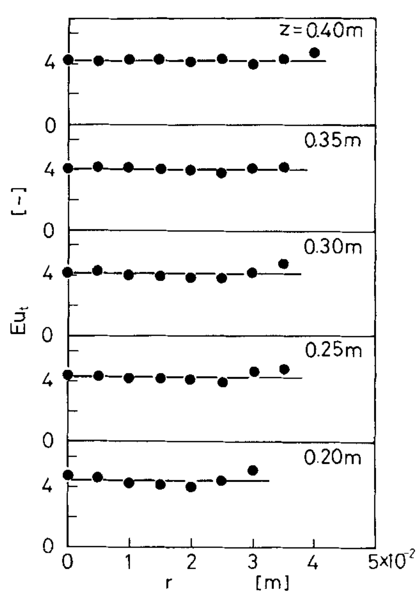

Figure 3. Constant Eu_t at Re = 11,000.

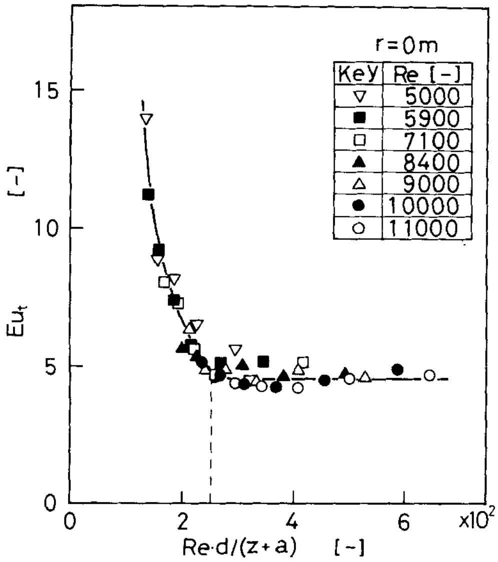

Figure 4. Effect of Re and $(z + a)/d$ on Eu_t at center axis.

tion at X_o can be expressed from Eq. 2 as follows:

$$\overline{p_s(X_o)u_k(X_o)} = \frac{\rho}{4\pi}\int_v \left[2\frac{\partial \bar{U}_i(X)}{\partial x_j}\frac{\partial \overline{u_j(X)u_k(X_o)}}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}\overline{u_i(X)u_j(X)u_k(X_o)}\right]\frac{dV(X)}{|X_o - X|} \quad (6)$$

When the local characteristic parameters, i.e., p_s', u', ℓ, and S, are used to make Eq. 6 a dimensionless form, the following relations are estimated to hold between X_o and arbitrary X,

$$p_s'(X) = f_p[(z+a)/d, r/(z+a)]\, p_s'(X_o) \quad (7a)$$

$$u'(X) = f_u[(z+a)/d, r/(z+a)]\, u'(X_o) \quad (7b)$$

$$S(X) = f_s[(z+a)/d, r/(z+a)]\, S(X_o) \quad (7c)$$

$$\ell(X) = f_\ell[(z+a)/d, r/(z+a)]\, \ell(X_o) \quad (7d)$$

These relations hold because the similarity of distributions to dimensionless coordinates $[(z + a)/d,\ r/(z + a)]$ is well known in a normal jet (Hinze, 1975), as shown by some examples in Figure 2. By using these local characteristic parameters, Eq. 6 is transformed into a dimensionless equation,

$$\overline{p_s(X_o)u_k(X_o)}^* = \frac{\overline{p_s(X_o)u_k(X_o)}}{p_s'(X_o)u'(X_o)} = \frac{1}{4\pi}\int_{v^*}\left\{2F_1\frac{\rho u'(X_o)^2}{p_s'(X_o)}\frac{S(X_o)l(X_o)}{u'(X_o)} \cdot \left[\frac{\partial \bar{U}_i(X)}{\partial x_j}\right]^* \frac{\partial \overline{u_j(X)^* u_k(X_o)^*}}{\partial x_i^*} + F_2\frac{\rho u'(X_o)^2}{p_s'(X_o)}\frac{\partial^2}{\partial x_i^* \partial x_j^*} \cdot \overline{u_i(X)^* u_j(X)^* u_k(X_o)^*}\right\}\frac{dV(X^*)}{|X_o - X|^*} \quad (8)$$

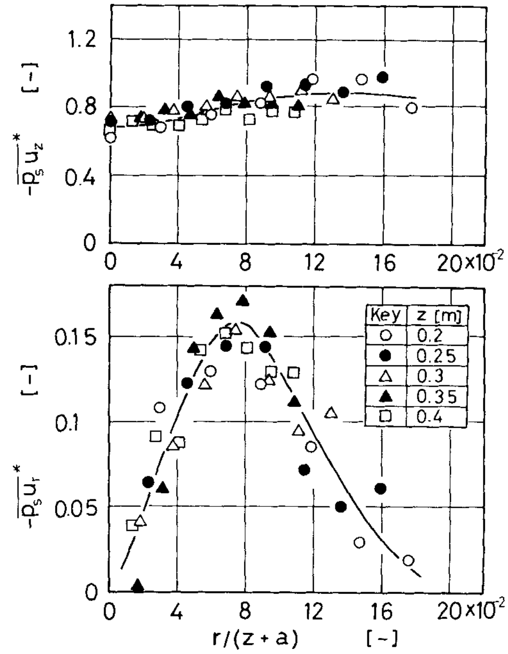

Figure 5. Similarity of $-\overline{p_s u_i}$ distributions at Re = 11,000.

where F_1 and F_2 are dimensionless functions composed of f_p, f_u, f_ℓ, and f_s. From Eq. 8, $\overline{p_s u_k}^*$ is estimated to become a function of Eu_t, $S\ell/u'$, and dimensionless coordinates:

$$\overline{p_s u_k}^* = G[Eu_t, S\ell/u', (z+a)/d, r/(z+a)] \quad (9)$$

In a fully turbulent jet region, both Eu_t and $S\ell/u'$ are considered to take constant values as mentioned above, and $\overline{p_s u_k}^*$ is estimated to be a function only of dimensionless coordinates. Figure 5 shows distributions of $-\overline{p_s u_r}^*$ and $-\overline{p_s u_z}^*$ vs. $r/(z + a)$, and they show the good similarity of distributions.

On the other hand, $\overline{p_s u_k}^*$ is estimated to change with Eu_t, which value in the transitional condition is different from that in the fully turbulent condition. Figure 6 shows a relationship between Eu_t and $-\overline{p_s u_z}^*$ at $r/(z + a) = 0$, and it becomes a monotonous decreasing function that starts from $Eu_t = 4.3$. Eu_t is considered to be an adequate factor to guess the degree of development of turbulence.

In the former model of a gradient type of diffusion for the

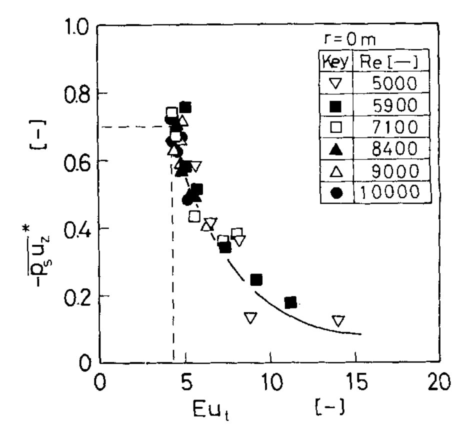

Figure 6. Effect of Eu_t on $-\overline{p_s u_z}^*$ at center axis.

static pressure-velocity correlation (Launder and Spalding, 1974), $p_s + \rho u_i u_i/2$ was regarded as the mechanical energy fluctuation and $\overline{(p_s + \rho u_i u_i/2)u_j}$ was assumed to be proportional to $\partial(\rho \overline{u_i u_i}/2)/\partial x_j$. In this case, the turbulent transport flux of mechanical energy was related only to the kinetic energy of turbulence, and the mean kinetic energy and the mean static pressure energy were not considered.

In this study, a gradient-type expression of turbulent transport of static pressure energy is investigated. The gradient of mean static pressure is assumed to be constant across the mixing length ℓ, and the static pressure energy contained in an eddy is assumed to be preserved across the mixing length ℓ. Hence, the following coefficients of turbulent diffusion for the static pressure energy are defined,

$$\overline{p_s u_r} = -\epsilon_r \frac{\partial \overline{P}_s}{\partial r} \quad (10)$$

$$\overline{p_s u_z} = -\epsilon_z \frac{\partial \overline{P}_s}{\partial z} \quad (11)$$

where $\overline{P}_s$ is the mean static pressure relative to the hydrostatic pressure. The gradients of the mean pressure in these equations can be expressed by the turbulent Navier-Stokes equation and are connected with stochastic quantities of velocity. Considering that the stochastic quantities of velocity whic are made dimensionless by $\overline{U}_{zm}$ show the similarity of distributions to $r/(z + a)$ in a normal jet (Hinze, 1975), distributions of $\overline{P}_s$ also are considered to be expressed in the same way, as follows:

$$\overline{P}_s = \rho \overline{U}_{zm}^2 g_p[r/(z + a)] \quad (12)$$

On the other hand, considering that distributions of u' and p'_s show the similarity of distributions in the same way, the following relation is expected:

$$\overline{p_s u_i} = \rho \overline{U}_{zm}^3 f_{pu}[r/(z + a)] \quad (13)$$

From Eqs. 12 and 13, the following dimensionless coefficients of turbulent diffusion ϵ_r^*, ϵ_z^* are expected to show the similarity of distribution to $r/(z + a)$:

$$\epsilon_r^* = \frac{\epsilon_r}{(z + a)\overline{U}_{zm}} \quad (14)$$

$$\epsilon_z^* = \frac{\epsilon_z}{(z + a)\overline{U}_{zm}} \quad (15)$$

Figure 7 shows distributions of ϵ_r^* and ϵ_z^* at Re = 11,000. Their values are reasonably positive in all measured positions, and their distributions show the similarity. Such ϵ_r and ϵ_z are considered to be adequate coefficients to estimate the values of $\overline{p_s u_r}$ and $\overline{p_s u_z}$.

In summary, the turbulent Euler number Eu_t is an important dimensionless factor for investigating the kinetic mechanism in the turbulent static pressure-velocity field, and it takes a constant value in the fully turbulent mixing region of jet flow. The similarity of distributions of the turbulent static pressure-velocity correlation $\overline{p_s u_i}$ depends on Eu_t, and the value of $\overline{p_s u_i}$ can be calculated by the gradient-type model of turbulent transport of static pressure energy in the fully turbulent condition of Eu_t = constant.

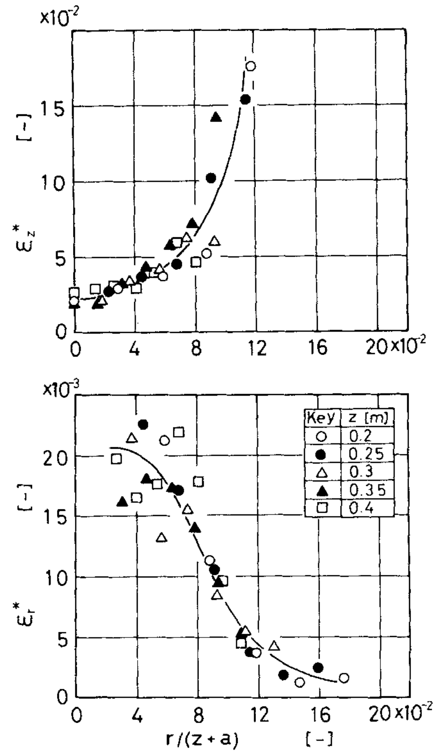

Figure 7. Similarity of ϵ_i distributions at Re = 11,000.

Notation

- a = distance to virtual source of jet from nozzle exit, m
- d = inner diameter of jet nozzle, m
- Eu_t = turbulent Euler number, $p'_s/\rho u'^2$
- e_{ij} = mean strain rate tensor, 1/s
- F = dimensionless function
- f = dimensionless function
- G = dimensionless function
- g = dimensionless function
- ℓ = mixing length, m
- P = pressure at pressure hole, Pa
- P_s = static pressure, Pa
- p_s = static pressure fluctuation, Pa
- p'_s = root mean square value of p_s, Pa
- Re = Reynolds number of jet, $U_o d/\nu$
- r = radius, m
- S = $(e_{ij}e_{ij})^{1/2}$, 1/s
- U = flow velocity, m/s
- U_i = velocity component, m/s
- $\underline{U}_o$ = velocity of primary stream at jet nozzle, m/s
- $\overline{U}_{zm}$ = maximum value of $\overline{U}_z$ at r = 0 m, m/s
- u_i = velocity fluctuation component, m/s
- u' = mean turbulent intensity, m/s
- X = coordinates vector, m
- x_i = Cartesian coordinates, m
- z = axial distance, m

Greek letters

- ϵ_i = coefficient of turbulent diffusion for static pressure, m^2/s
- θ = angle between OM axis of probe and velocity vector, rad
- ν = kinematic viscosity, m^2/s
- ρ = density, kg/m^3

Subscripts and superscripts

$i, j, k = r, \theta, z$ (cylindrical polar coordinates), or 1, 2, 3 (Cartesian coordinates)

$*$ = dimensionless quantity

$-$ = time-mean value

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Manuscript received Feb. 26, 1986, and revision received June 18, 1986.